



# POSTAL BOOK PACKAGE 2027

## INSTRUMENTATION ENGINEERING

### OBJECTIVE PRACTICE SETS VOLUME - I

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# **ELECTRICAL CIRCUITS**

**OBJECTIVE PRACTICE SETS**

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# 1

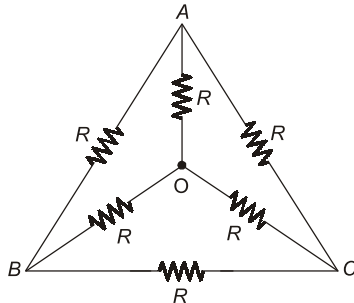
## CHAPTER

# Circuit Element and Energy Sources

### MCQ and NAT Questions

- Q.1** The equivalent star impedance of a balanced delta connected load of value  $6 + j9 \Omega$  is given by  
 (a)  $9 + j6 \Omega$       (b)  $2 + j3 \Omega$   
 (c)  $18 + j27 \Omega$       (d)  $6 - j9 \Omega$

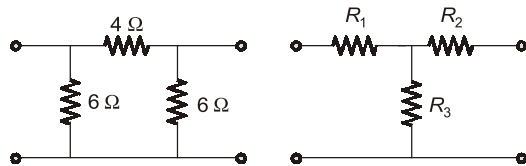
- Q.2** The effective resistance between the terminals  $A$  and  $B$  in the circuit shown in the figure is



- (a)  $R$       (b)  $R - 1$   
 (c)  $\frac{R}{2}$       (d)  $\frac{6}{11}R$

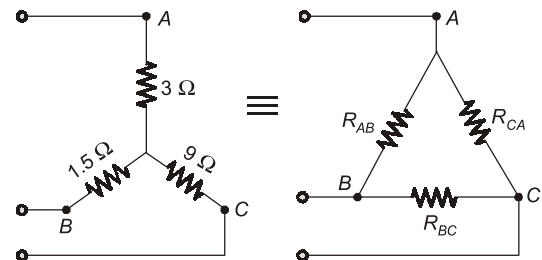
- Q.3** A network contains only independent current sources and resistors. If the values of all resistors are doubled, the values of the node voltages  
 (a) will become half  
 (b) will remain unchanged  
 (c) will become double  
 (d) cannot be determined unless the circuit configuration and the values of the resistors are known

- Q.4** The value of  $R_1$ ,  $R_2$  and  $R_3$  of the equivalent 'T' network for the given  $\pi$  network will be such that



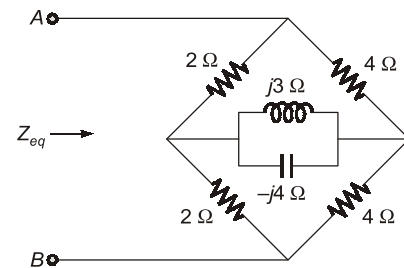
- (a)  $2.25 \Omega$        $1.5 \Omega$        $1.5 \Omega$   
 (b)  $1.5 \Omega$        $1.5 \Omega$        $2.25 \Omega$   
 (c)  $2.25 \Omega$        $1.5 \Omega$        $2.25 \Omega$   
 (d)  $1.5 \Omega$        $2.25 \Omega$        $1.5 \Omega$

- Q.5** For the equivalent figure circuit shown in the given figure, the values of  $R_{AB}$  and  $R_{BC}$  are respectively



- (a)  $5 \Omega$  and  $15 \Omega$       (b)  $15 \Omega$  and  $30 \Omega$   
 (c)  $30 \Omega$  and  $5 \Omega$       (d)  $20 \Omega$  and  $35 \Omega$

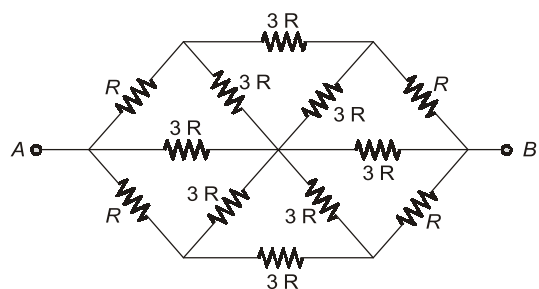
- Q.6** In the circuit of figure. The equivalent impedance seen across terminals  $A, B$  is \_\_\_\_\_  $\Omega$ .



- Q.7** If each branch of a delta circuit has impedance  $Z/\sqrt{3}$  then, each branch of the equivalent Y circuit has impedance.

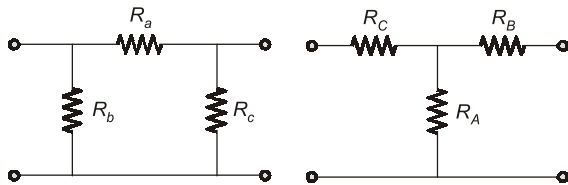
- (a)  $\frac{Z}{\sqrt{3}}$       (b)  $\frac{Z}{3\sqrt{3}}$   
 (c)  $3\sqrt{3}Z$       (d)  $Z/3$

- Q.8** The equivalent resistance between terminals  $A$  and  $B$  for the circuit shown is:



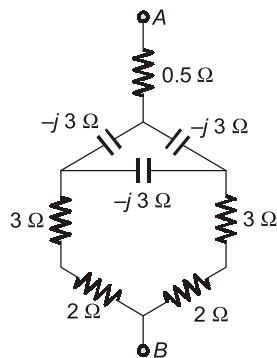
- (a)  $R$  (b)  $\frac{R}{3}$   
(c)  $\frac{3R}{2}$  (d)  $2R$

**Q.9** Consider a delta connection of resistors and its equivalent star connection as shown below. If all elements of delta connection are scaled by a factor of  $K$ ,  $K > 0$ , the elements of the corresponding star equivalent will be scaled by a factor of



- (a)  $K^2$  (b)  $K$   
(c)  $1/K$  (d)  $\sqrt{K}$

**Q.10** For the circuit shown, the impedance between terminals A and B is:



- (a)  $9 - j3 \Omega$  (b)  $3 - j1.5 \Omega$   
(c)  $j1.5 \Omega$  (d)  $0 \Omega$

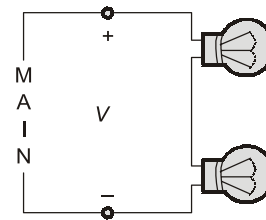
**Q.11** A lamp rated at 10 watts, 50 volts is proposed to be used in 110 volts, system. The wattage and resistance of the resistor to be connected in series with the lamp should be

- (a) 15 watt, 350 ohms  
(b) 10 watts, 250 ohms  
(c) 12 watts, 300 ohms  
(d) 15 watts, 250 ohms

**Q.12** The equivalent resistance of four resistors joined in parallel is 20 ohms. The currents flowing through them are 0.6, 0.3, 0.2 and 0.1 amp. The lowest value resistor is of

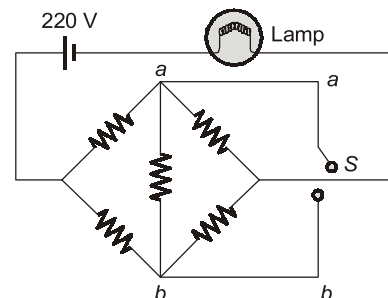
- (a) 240 ohms (b) 120 ohms  
(c) 80 ohms (d) 40 ohms

**Q.13** The incandescent bulbs rated respectively as  $P_1$  and  $P_2$  for operation at a specified mains voltage are connected in series across the mains as shown in the below figure. Then the total power supplied by the mains to the two bulbs are



- (a)  $\frac{P_1 P_2}{P_1 + P_2}$  (b)  $\sqrt{P_1^2 + P_2^2}$   
(c)  $(P_1 + P_2)$  (d)  $\sqrt{P_1 \times P_2}$

**Q.14** All resistances in the circuit in figure are of  $R$  ohms each. The switch is initially open. What happens to the lamp's intensity when the switch is closed?



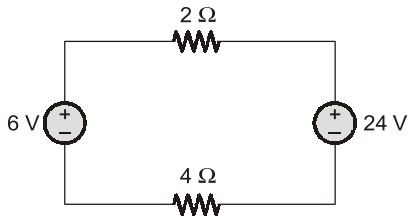
- (a) increases  
(b) decreases  
(c) remains the same  
(d) answer depends on the value of  $R$

**Q.15** A practical current source is usually represented by

- (a) a resistance in series with an ideal current source.  
(b) a resistance in parallel with an ideal current source.  
(c) a resistance in parallel with an ideal voltage source.  
(d) none of these

## Multiple Select Questions (MSQs)

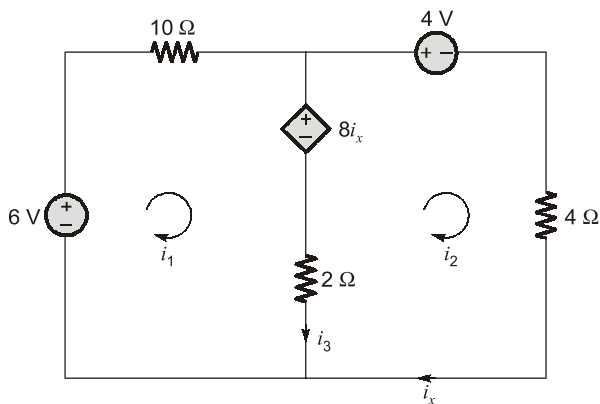
Q.24 For the circuit shown below :



Which of the following is correct?

- (a) Power delivered by 24 V source is 72 W.
- (b) Power absorbed by 4 Ω resistance is 36 W.
- (c) Power delivered by 24 V source is 0 W.
- (d) Power delivered by 6 V source is 18 W.

Q.25 For the circuit shown below :



Which of the following is correct?

- (a)  $i_1 = -1$  A
- (b)  $i_2 = -3$  A
- (c)  $i_3 = 4$  A
- (d)  $i_x = 3$  A

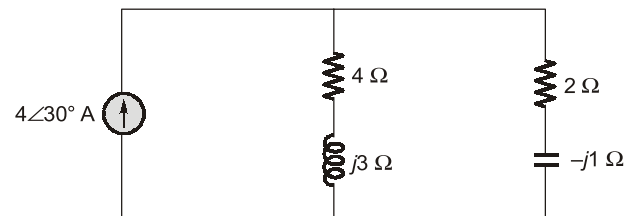
Q.26 A series circuit containing passive elements has the following current and applied voltage :

$$V = A \sin\left(\omega t + \frac{\pi}{4}\right) \text{ V} \quad ; \quad i = B \sin\left(\omega t - \frac{\pi}{6}\right) \text{ A}$$

The circuit elements :

- (a) may be resistance and inductance.
- (b) may be inductance, capacitance and resistance.
- (c) may be resistance.
- (d) may be resistance and capacitance.

Q.27 Consider the circuit shown in the figure below :



Which of the following statement is correct?

- (a) The power supplied by the current source is 14 W.
- (b) The average power absorbed by the capacitor is 0 W.
- (c) The average power absorbed by the 4 Ω resistor is 4 W.
- (d) The average power absorbed by the 2 Ω resistor is 11 W.

■■■■

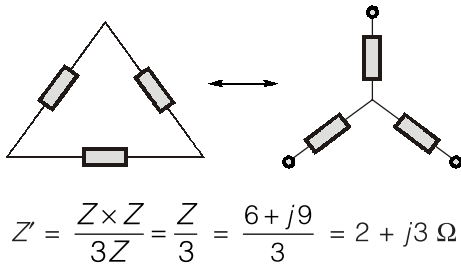
**Answers      Circuit Element and Energy Sources**

1.	(b)	2.	(c)	3.	(c)	4.	(b)	5.	(a)	6.	(2.67)	7.	(b)
8.	(c)	9.	(b)	10.	(b)	11.	(c)	12.	(d)	13.	(a)	14.	(c)
15.	(b)	16.	(c)	17.	(a)	18.	(a)	19.	(d)	20.	(d)	21.	(c)
22.	(a)	23.	(a)	24.	(a, b)	25.	(a, d)	26.	(a, b)	27.	(a, b, c)		

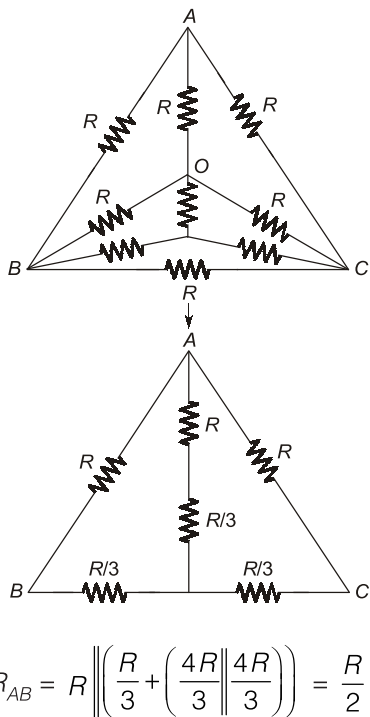
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**Explanations** Circuit Element and Energy Sources

1. (b)



2. (c)



3. (c)

Since the network contains only independent current sources, so changing resistors in the same proportion the current through each branch will remain same but node voltages will change in the same proportion. Hence, doubling all resistors, node voltages will be doubled.

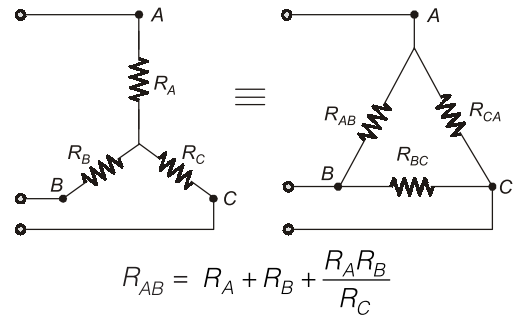
4. (b)

$$R_1 = \frac{4 \times 6}{4 + 6 + 6} = \frac{24}{16} = 1.5 \Omega$$

$$R_2 = \frac{6 \times 4}{16} = 1.5 \Omega$$

$$R_3 = \frac{6 \times 6}{16} = 2.25 \Omega$$

5. (a)



$$R_{AB} = 3 + 1.5 + \frac{3 \times 1.5}{9}$$

$$= 3 + 1.5 + 0.5 = 5 \Omega$$

$$R_{BC} = 9 + 1.5 + \frac{9 \times 1.5}{3}$$

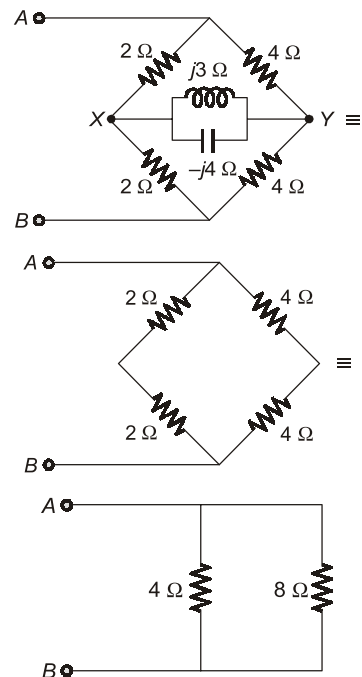
$$= 9 + 1.5 + 4.5 = 15 \Omega$$

$$R_{CA} = R_A + R_C + \frac{R_A R_C}{R_B}$$

$$= 3 + 9 + \frac{3 \times 9}{1.5} = 30 \Omega$$

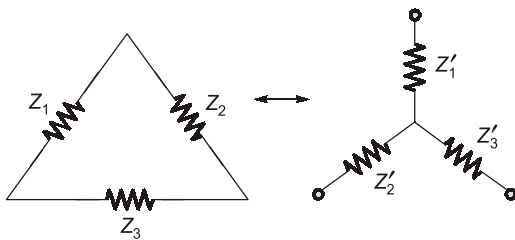
6. Sol.

The above circuit is a Wheatstone bridge circuit, thus no current will flow through branch XY.



$$Z_{eq} = 8 \parallel 4 = \frac{8 \times 4}{12} = \frac{8}{3} = 2.67 \Omega$$

7. (b)



$$Z'_1 = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

$$Z'_2 = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$$

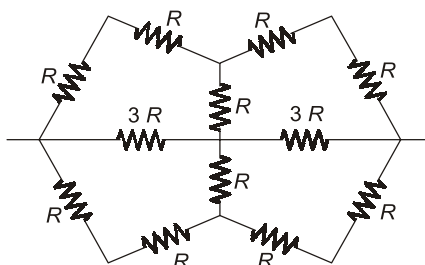
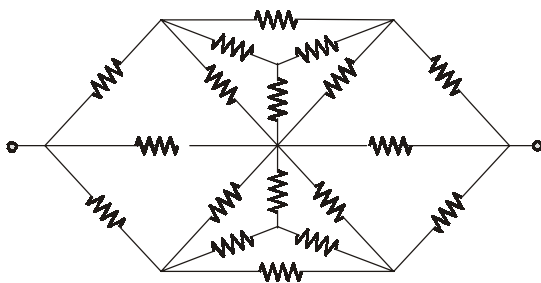
$$Z'_3 = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z'_1 = \frac{\frac{Z}{\sqrt{3}} \times \frac{Z}{\sqrt{3}}}{\frac{Z}{\sqrt{3}} + \frac{Z}{\sqrt{3}} + \frac{Z}{\sqrt{3}}}$$

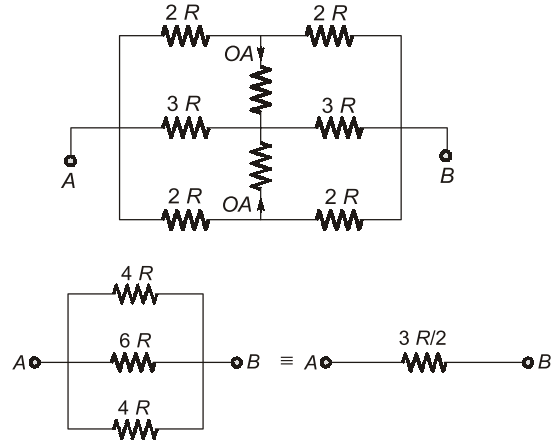
$$= \frac{Z^2}{\frac{(\sqrt{3})^2}{3Z}} = \frac{Z^2 \sqrt{3}}{3Z \times 3}$$

$$\Rightarrow Z'_1 = \frac{Z}{3\sqrt{3}}$$

8. (c)



9. (b)

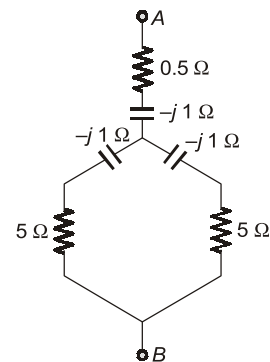


$$R_A = \frac{R_b R_c}{R_a + R_b + R_c} = K \cdot \left( \frac{R_c R_b}{R_a + R_b + R_c} \right)$$

$$R_B = \frac{R_a R_c}{R_a + R_b + R_c} = K \cdot \left( \frac{R_a R_c}{R_a + R_b + R_c} \right)$$

$$R_C = \frac{R_a \cdot R_b}{R_a + R_b + R_c} = \frac{K^2 \cdot R_a R_b}{K(R_a + R_b + R_c)} = \frac{K \cdot R_a R_b}{R_a + R_b + R_c}$$

10. (b)

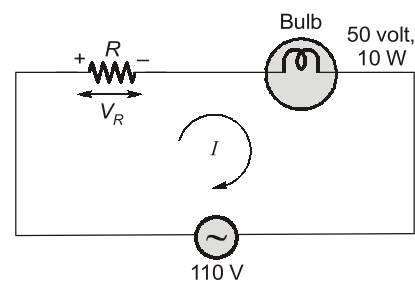


$$Z_{AB} = [(5-j) \parallel (5-j)] + (0.5-j)$$

$$= [2.5 - j/2] + (0.5-j)$$

$$= (3 - j 1.5) \Omega$$

11. (c)



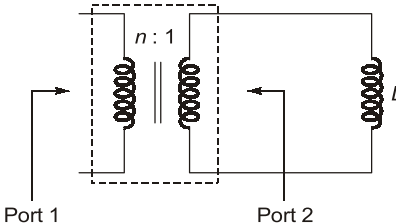
# **ELECTRICAL MACHINES**

**OBJECTIVE PRACTICE SETS**

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## Transformers

## MCQ and NAT Questions

- Q.1** The power transformer is a  
 (a) constant voltage device  
 (b) constant main flux device  
 (c) constant current device  
 (d) constant power device
- Q.2**  $P_i$  = core loss,  $P_c$  = copper loss. A transformer has maximum efficiency when  
 (a)  $P_i/P_c = 2$       (b)  $P_i/P_c = 15$   
 (c)  $P_i/P_c = 1$       (d)  $P_i/P_c = 0.5$
- Q.3** For a single-phase transformer,  $r_e$  = total equivalent resistance,  $x_e$  = total equivalent leakage reactance,  $P_c$  = core loss. The load current at which maximum efficiency occurs is  
 (a)  $\frac{P_c}{x_e}$       (b)  $\sqrt{\frac{P_c}{x_e}}$   
 (c)  $\frac{P_c}{r_e}$       (d)  $\sqrt{\frac{P_c}{r_e}}$
- Q.4** If the iron core of a transformer is replaced by an air core, then the hysteresis losses in the transformer will  
 (a) increase      (b) decrease  
 (c) remain unchanged      (d) become zero
- Q.5** A 2 kVA transformer has iron loss of 150 Watts and full-load copper loss of 250 Watts. The maximum efficiency of the transformer would occur when the total loss is  
 (a) 500 W      (b) 400 W  
 (c) 300 W      (d) 275 W
- Q.6** A 1-phase 250/500 V transformer gave the following result. Open-circuit test 250 V, 1 A, 80 W on l.v. side what is the power factor?  
 (a) 0.24      (b) 0.32  
 (c) 0.28      (d) 0.30
- Q.7** Which of the following is not true relating to an ideal transformer?  
 (a) The iron loss in an ideal transformer is zero  
 (b) The winding resistance has a zero value  
 (c) The leakage reactance has a non-zero value  
 (d) The magnetizing current is zero
- Q.8** Two transformers with identical voltage ratings are working in parallel to supply a common load. The impedance of one transformer is higher compared to that of the other. The load sharing between the two transformers will  
 (a) be proportional to their impedances.  
 (b) be independent of their impedances.  
 (c) be inversely proportional to their respective impedances.  
 (d) depend on the resistance to leakage reactance ratio of each transformer.
- Q.9** A 20 kVA, 2000/200 V transformer has name plate leakage impedance of 8% what voltage must be applied on the HV side to circulate full load current with the LV shorted:  
 (a) 200 V      (b) 2000 V  
 (c) 160 V      (d) 240 V
- Q.10** If an ideal transformer has an inductive load element at port 2 as shown in the figure below, the equivalent inductance at port 1 is  
  
 (a)  $nL$       (b)  $n^2L$   
 (c)  $\frac{n}{L}$       (d)  $\frac{n^2}{L}$
- Q.11** A transformer operates most efficiently at  $3/4^{\text{th}}$  full-load. Its iron loss ( $P_i$ ) and full-load copper loss ( $P_c$ ) are related as:  
 (a)  $P_i/P_c = 16/9$       (b)  $P_i/P_c = 4/3$   
 (c)  $P_i/P_c = 3/4$       (d)  $P_i/P_c = 9/16$

**Q.12** The efficiency of transformer during no load and short-circuit test is

- (a) maximum (b) 50%  
(c) zero (d) none of the above

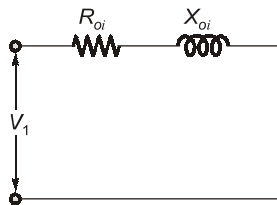
**Q.13** During open-circuit test in a transformer the wattmeter used is of

- (a) low p.f. wattmeter type  
(b) high p.f. wattmeter type  
(c) medium p.f. wattmeter type  
(d) very high p.f. wattmeter type

**Q.14** Find the reactive component of no-load current for a 440/220 V single-phase transformer if power input to H.V. winding is 110 W. The low voltage winding is kept open. The p.f. of no-load current is 0.6 lagging.

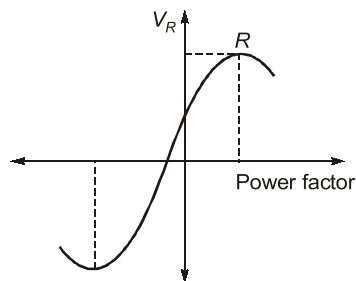
- (a) 0.578 A (b) 0.80 A  
(c) 0.33 A (d) 0.427 A

**Q.15** At which condition of the transformer the equivalent circuit will be as shown in the below figure?



- (a) Under short circuit (b) Under open circuit  
(c) Under no load (d) Under rated load

**Q.16** A 2300/230 V single phase transformer has resistance and reactance 0.05 p.u. and 0.5 p.u. respectively. Voltage regulation versus load power factor plot is shown below:



At point *R*, power factor of load is

- (a) 0.995 lagging (b) 0.0995 lagging  
(c) 0.0995 leading (d) 0.995 leading

**Q.17** A 50 kVA transformer has a core loss of 600 W and a full-load copper loss of 900 W. The proportion of full load at maximum efficiency is

- (a) 91.03% (b) 72.16%  
(c) 81.64% (d) 68.45%

**Q.18** The full load copper loss and core loss of a transformer are 3200 W and 1500 W respectively. The total loss (neglecting stray losses) at  $3/4^{\text{th}}$  of full load is \_\_\_\_\_ W.

**Q.19** A 300 kVA transformer has a core loss of 1.5 kW and a full load copper loss of 4.5 kW, the maximum efficiency of the transformer is \_\_\_\_\_ %.

**Q.20** Which of these following transformers for per unit loading will reach full load first?

- (a) 1000 kVA,  $Z = 2 \Omega$   
(b) 5000 kVA,  $Z = 4 \Omega$   
(c) 2000 kVA,  $Z = 1.5 \Omega$   
(d) 4000 kVA,  $Z = 1 \Omega$

**Q.21** A transformer has percentage reactance of 4% and percentage resistance of 2%. The voltage regulations at pf 0.6 lead and 0.6 lag respectively are

- (a) 4.4%, -2% (b) -2.4%, 0.8%  
(c) -2%, 4.4% (d) 0.8%, -2.4%

**Q.22** A 200 V/100 V, 50 Hz transformer is to be excited at 40 Hz from 100 V side. For the exciting current to be the same, the applied voltage should be

- (a) 150 V (b) 80 V  
(c) 100 V (d) 125 V

**Q.23** Eddy current losses in transformer cores can be reduced by the use of

1. Solid cores
2. Laminated cores
3. Ferrites

Select the correct answer using the codes given below:

- (a) 2 and 3 only (b) 1 and 2 only  
(c) 1 and 3 only (d) 1, 2 and 3

**Q.24** No load current in a transformer

- (a) lags the applied voltage by  $90^\circ$   
(b) lags the applied voltage by somewhat less than  $90^\circ$   
(c) leads the applied voltage by  $90^\circ$   
(d) leads the applied voltages by somewhat less than  $90^\circ$

**Q.25** A 100 kVA, 1000 V/400 V, 1- $\phi$  transformer when excited at rated voltage on *h.v.* side, draws a no load current of 3.0 A at 0.75 lagging power factor. If it is excited from the *l.v.* side at rated voltage, the no load current and power factor will be respectively,

- (a) 7.5 A, 0.6 lag (b) 7.5 A, 0.75 lag  
(c) 6.5 A, 0.70 lag (d) 6.5 A, 0.75 lag

Answers		Transformers				
1. (b)	2. (c)	3. (d)	4. (d)	5. (c)	6. (b)	7. (c)
8. (c)	9. (c)	10. (b)	11. (d)	12. (c)	13. (a)	14. (c)
15. (a)	16. (b)	17. (c)	18. (3300)	19. (98.29)	20. (a)	21. (c)
22. (b)	23. (a)	24. (b)	25. (b)	26. (d)	27. (18)	28. (d)
29. (c)	30. (d)	31. (15.65)	32. (6.02)	33. (b)	34. (d)	35. (d)
36. (a)	37. (3.556)	38. (95.92)	39. (a)	40. (1.2)	41. (b)	42. (a)
43. (c)	44. (151.95)	45. (b)	46. (a)	47. (1.44)	48. (b)	49. (c)
50. (c)	51. (a)	52. (c)	53. (2.4)	54. (23.61)	55. (a, b)	56. (a, c)
57. (b, c, d)	58. (a, b, c)					

**Explanations**      **Transformers**

**1. (b)**

Transformer is a static device which cannot change frequency, but can change voltage and current level of the system. Practically, transformer is a constant flux machine. However due to losses in transformer, power at both sides may vary. Hence (b) is better option.

**2. (c)**

In a transformer, condition for maximum efficiency is copper loss or variable loss = iron loss  
 Thus,  $P_i = P_c$   
 or  $\frac{P_i}{P_c} = 1$

**3. (d)**

At  $\eta_{max}$ ,  
 Copper loss = Iron loss  
 or,  $I_m^2 r_e = P_c$   
 or,  $I_m = \sqrt{\frac{P_c}{r_e}}$

**4. (d)**

Air-core means non-iron core so there will be no hysteresis losses.

**5. (c)**

For maximum efficiency at full load  
 $P_{cu} = P_i = 150 \text{ W}$   
 Total loss =  $P_i + P_{cu} = 150 + 150 = 300 \text{ W}$

**6. (b)**

Given:  $V_{NL} = 250 \text{ V}$  ;  $I_{NL} = 1 \text{ A}$  ;  $P_{NL} = 80 \text{ W}$   
 For OC test:  $P_{NL} = V_{NL} I_{NL} \cdot \cos \phi$   
 $\cos \phi = \text{pf on LV side}$   
 $= \frac{80}{250 \times 1} = 0.32$

**7. (c)**

In an ideal transformer there is no-loss (either in winding or core). The leakage reactance is also zero (as no-voltage drop).  
 Magnetizing current is zero due to infinite permeability of core.

**8. (c)**

$$\text{kVA shared} \propto \frac{1}{\text{leakage impedance}}$$

**9. (c)**

Given,  $\frac{Z_{HV(\Omega)} I_{HV(\text{rated})}}{V_{HV(\text{rated})}} = 0.08$   
 $Z_{HV(\Omega)} \cdot I_{HV(\text{rated})} = 0.08 V_{HV(\text{rated})}$   
 $V_{SC} = 0.08 \times 2000 = 160 \text{ V}$

**10. (b)**

In an ideal transformer,  
 $Z \propto N^2$   
 Given, on port 2:  
 $Z_2 = \omega L$  i.e.  $Z_2 \propto L$   
 $\frac{N_1}{N_2} = n : 1 = n$

$$\frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2}\right)^2$$

$$Z_1 = (Z_2) [n]^2$$

$$Z_1 = (\omega L)(n^2)$$

$$X_{L1} = \text{equivalent inductance at port 1:}$$

$$= \frac{Z_1}{\omega} = n^2 L$$

**11. (d)**

For maximum efficiency,

$$P_i = x^2 P_{cu}$$

$$\Rightarrow P_i = P_{cu} \text{ (for full load)}$$

For 3/4<sup>th</sup> full load,

$$P_i = \left(\frac{3}{4}\right)^2 P_{cu}$$

$$\frac{P_i}{P_{cu}} = \frac{9}{16}$$

**12. (c)**

SC test is carried out to determine the full load copper loss and OC test or NL test to determine core loss. During these tests, power required is used to determine the losses and efficiency will be zero.

**13. (a)**

During open circuit test or no load test, it is recommended to use low power factor wattmeter. This is because NL power factor is very low and thus less deflecting torque ( $T_d \propto I_1 I_2 \cos \phi$ ) will be there. Hence LPF wattmeters employed for OC test. On the other hand, for short circuit test, high pf wattmeters are employed.

**14. (c)**

$$\text{Given, } P_0 = 110 \text{ W, } V_0 = 440 \text{ V}$$

$$\cos \phi_0 = 0.6 \text{ (lag)}$$

$$\therefore \text{At NL test, } P_0 = V_0 I_0 \cos \phi_0$$

No load current,

$$I_0 = \frac{110}{440 \times 0.6} = 0.4166 \text{ A}$$

Let reactive component of no-load current =  $I_m$ 

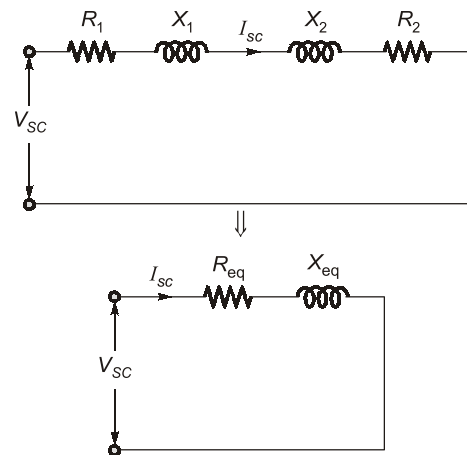
$$I_m = I_0 \sin \phi_0$$

$$= 0.4166 \times 0.8 = 0.33 \text{ A}$$

**15. (a)**

- SC test is conducted at H.V. side to find out the total resistance and reactance of transformer.

- Equivalent circuit during S.C. condition.

**16. (b)**

Point R is corresponding to maximum voltage regulation.

For maximum voltage regulation load power factor is equal to

$$\cos \phi = \frac{R}{Z}$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{0.05^2 + 0.5^2}$$

Power factor,

$$\cos \phi = 0.0995 \text{ lagging}$$

**17. (c)**

Proportion of full load at maximum efficiency,

$$x = \sqrt{\frac{P_i}{P_{cu(fl)}}} = \sqrt{\frac{600}{900}}$$

$$= 0.8164 = 81.64\%$$

**18. (3300)**

$$\text{Given, } P_i = 1500 \text{ W, } P_{CFL} = 3200 \text{ W}$$

At  $\frac{3}{4}$ th of full load,

$$\text{Copper losses} = P_C' = 3200 \times \left(\frac{3}{4}\right)^2 = 1800 \text{ W}$$

But iron losses or core losses will be same

Total losses at  $\frac{3}{4}$ th of full load

$$= P_C' + P_i$$

$$= 1800 + 1500 = 3300 \text{ W}$$

**19. (98.29)**

Maximum efficiency occurs at unity power factor and when copper loss is equal to the iron loss kVA rating at maximum efficiency

# **ELECTRICITY AND MAGNETISM**

**OBJECTIVE PRACTICE SETS**

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## Vector Analysis

## MCQ and NAT Questions

**Q.1** If  $\vec{G} = 15r\hat{a}_\phi$  then  $\oint \vec{G} \cdot d\vec{l}$  over the circular path

$r = 2 \text{ m}$ ,  $\theta = 30^\circ$ ,  $0 < \phi < 2\pi$  is

- (a)  $120\pi$  (b) 120  
(c)  $60\pi$  (d) 60

**Q.2** Which of the following is true?

- (a)  $\text{Curl}(\vec{A} \cdot \vec{B}) = \text{Curl } \vec{A} + \text{Curl } \vec{B}$   
(b)  $\text{Div}(\vec{A} \cdot \vec{B}) = \text{Div } \vec{A} \cdot \text{Div } \vec{B}$   
(c)  $\text{Div}(\text{Curl } \vec{A}) = 0$   
(d)  $\text{Div}(\text{Curl } \vec{A}) = \Delta \cdot \vec{A}$

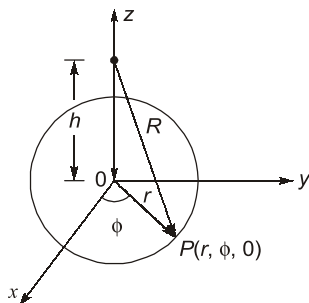
**Q.3** Which of the following equations is correct?

- $\hat{a}_x \times \hat{a}_x = |\hat{a}_x|^2$
  - $(\hat{a}_x \times \hat{a}_y) + (\hat{a}_y \times \hat{a}_x) = 0$
  - $\hat{a}_x \times (\hat{a}_y \times \hat{a}_z) = \hat{a}_x \times (\hat{a}_z \times \hat{a}_y)$
  - $\hat{a}_r \cdot \hat{a}_\theta + \hat{a}_\theta \cdot \hat{a}_r = 0$
- (a) 1 and 2 only (b) 2 and 3 only  
(c) 1 and 3 only (d) 2 and 4 only

**Q.4** Laplacian of a scalar function  $V$  is

- (a) Gradient of  $V$   
(b) Divergence of  $V$   
(c) Gradient of the gradient of  $V$   
(d) Divergence of the gradient of  $V$

**Q.5** The unit vector  $\vec{a}_r$  which points from  $z = h$  on the  $z$ -axis towards  $(r, \phi, 0)$  in cylindrical co-ordinates as shown below is given by



- (a)  $\frac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$  (b)  $\frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$   
(c)  $\frac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$  (d)  $\frac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$

**Q.6** Which of the following statements is not true of a phasor?

- (a) It may be a scalar or a vector.  
(b) It is a time dependent quantity.  
(c) It is a complex quantity.  
(d) All are true.

**Q.7** The maximum space rate of change of the function which is in increasing direction of the function is known as

- (a) curl of the vector function  
(b) gradient of the scalar function  
(c) divergence of the vector function  
(d) Stokes theorem

**Q.8 Assertion (A):** Divergence of a vector function  $\vec{A}$  at each point gives the rate per unit volume at which the physical entity is issuing from that point.

**Reason (R):** If some physical entity is generated or absorbed within a certain region of the field, then that region is known as source or sink respectively and if there are no sources or sinks in the field, the net outflow of the incompressible physical entity over any part of the region is zero. However, the net outflow is said to be positive, if the total strength of the sources are greater than the total strength of sink and vice-versa.

- (a) Both A and R are true and R is a correct explanation of A.  
(b) Both A and R are true but R is not a correct explanation of A.  
(c) A is true but R is false.  
(d) A is false but R is true.

**Q.9** Given a vector field  $\vec{F}$ . The Stoke's theorem states that,

(a)  $\oint \vec{F} \cdot d\vec{l} = \iint (\vec{\nabla} \times \vec{F}) \cdot d\vec{s}$

(b)  $\oint \vec{F} \times d\vec{l} = \iint (\vec{\nabla} \cdot \vec{F}) d\vec{s}$

(c)  $\int \vec{F} \cdot d\vec{l} = \iint (\vec{\nabla} \times \vec{F}) \cdot d\vec{s}$

(d)  $\int \vec{F} \times d\vec{l} = \iint (\vec{\nabla} \cdot \vec{F}) d\vec{s}$

**Q.10** The vector  $\vec{A}$  directed from (2, -4, 1) to (0, -2, 0) in Cartesian coordinates is given by

(a)  $-2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$  (b)  $-2\vec{a}_x + 2\vec{a}_y - \vec{a}_z$

(c)  $-\vec{a}_x - 2\vec{a}_y + 2\vec{a}_z$  (d)  $\vec{a}_x - 2\vec{a}_y - \vec{a}_z$

**Q.11** The vector field given by

$$\vec{A} = yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z$$
 is

(a) rotational and solenoidal

(b) rotational but not solenoidal

(c) irrotational and solenoidal

(d) irrotational but not solenoidal

**Q.12** If  $\vec{A} = \frac{\vec{a}_x}{\sqrt{x^2 + y^2}}$ , then the value of  $\nabla \cdot \vec{A}$  at (2, 2, 0)

will be

(a) -0.0884 (b) 0.0264

(c) -0.0356 (d) 0.0542

**Q.13** If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ , then the value of  $\vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k})$  is

(a)  $\vec{r}$  (b)  $2\vec{r}$

(c)  $3\vec{r}$  (d)  $6\vec{r}$

**Q.14** What is the value of constant  $b$  so that the vector

$$\vec{V} = (x + 3y)\vec{i} + (y - 2x)\vec{j} + (x + bz)\vec{k}$$

is solenoidal?

(a) 2 (b) -1

(c) 3 (d) -2

**Q.15 Assertion (A):** The Gauss's divergence theorem permits us to express certain integrals by means of surface integrals.

**Reason (R):** Gauss's divergence theorem states that "the surface integral of the curl of a vector field taken over any surface  $s$  is equal to the line integral of the vector field around the closed periphery (contour) of the surface."

(a) Both A and R are true and R is a correct explanation of A.

(b) Both A and R are true but R is not a correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

**Q.16** Which of the following option is not correct?

(a) A vector field  $\vec{A}$  is solenoid, if  $\nabla \cdot \vec{A} = 0$ (b) A vector field  $\vec{A}$  is irrotational, if  $\nabla \times \vec{A} = 0$ (c) A vector field  $V$  is harmonics, if  $\nabla^2 V \neq 0$ 

(d) All options are correct

**Q.17** Which of the following statements is not true regarding vector algebra?

(a) Dot product of like unit vector is unity.

(b) Dot product of unlike unit vector is zero.

(c) Cross product of two like unit vectors is a third unit vector having positive sign for normal rotation and negative for reverse rotation.

(d) All the above statements are true.

**Q.18** A rigid body is rotating with an angular velocity of

 $\omega$  where,  $\vec{\omega} = \omega_x\vec{i} + \omega_y\vec{j} + \omega_z\vec{k}$  and  $v$  is the linevelocity. If  $\vec{r}$  is the position vector given by $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ , then the value of  $\text{curl } \vec{v}$  will be equal to

(a)  $1/2 \omega$  (b)  $\omega$

(c)  $1/3 \omega$  (d)  $2 \omega$

**Q.19** Which of the following identity is not true?

(a)  $\vec{A}(\vec{B} \cdot \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

(b)  $\nabla \cdot (\nabla \times \vec{A}) = 0$

(c)  $\nabla \times \nabla \phi \neq 0$

(d) None of the above

**Q.20** The value of divergence of a vector quantity

 $\vec{A} = 4xy\hat{a}_x + xz\hat{a}_y + xyz\hat{a}_z$  at a point  $P(1, -2, 3)$  will be

(a) 6 (b) -16

(c) -10 (d) 12

**Q.21** Laplace equation in cylindrical coordinates is given by

(a)  $\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \left( \frac{\partial^2 V}{\partial \phi^2} \right) + \frac{\partial^2 V}{\partial z^2} = 0$

(b)  $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

**Answers**    **Vector Analysis**

1. (c)    2. (c)    3. (d)    4. (d)    5. (b)    6. (a)    7. (b)    8. (a)    9. (a)  
 10. (b)    11. (c)    12. (a)    13. (b)    14. (d)    15. (c)    16. (c)    17. (c)    18. (d)  
 19. (c)    20. (c)    21. (a)    22. (62.83)    23. (5.14)    24. (-1.15)    25. (d)    26. (a)    27. (3.75)  
 28. (a)    29. (d)    30. (129.43)    31. (c)    32. (d)    33. (b)    34. (2)    35. (1244)  
 36. (b)    37. (a)    38. (5)    39. (c)    40. (a)    41. (d)    42. (a)    43. (d)    44. (b)  
 45. (3)    46. (0.33)    47. (0.5)    48. (0.5)    49. (a,c,d)    50. (b,d)    51. (b,c)    52. (c,d)    53. (a,d)  
 54. (b,c,d)    55. (b,c)

**Explanations**    **Vector Analysis****1. (c)**

For spherical coordinate systems,

$$\begin{aligned}\vec{dl} &= r \sin\theta d\phi \hat{a}_\phi \\ \oint \vec{G} \cdot \vec{dl} &= \int_0^{2\pi} 15r \hat{a}_\phi \cdot r \sin\theta d\phi \hat{a}_\phi \\ &= 15 \cdot r^2 \cdot \sin\theta (2\pi) \\ &= 15 \cdot (2)^2 \times \sin 30^\circ (2\pi) \\ \oint \vec{G} \cdot \vec{dl} &= 60\pi\end{aligned}$$

**2. (c)**Divergence (Curl  $\vec{A}$ ) = 0**3. (d)**

$$(\hat{a}_x \times \hat{a}_x) = 0$$

Since cross product with same vector is zero because  $\theta = 0$  so  $\sin\theta = 0$ 

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$\hat{a}_y \times \hat{a}_x = -\hat{a}_z$$

$$(\hat{a}_x \times \hat{a}_y) + (\hat{a}_y \times \hat{a}_x) = \hat{a}_z + (-\hat{a}_z) = 0$$

**4. (d)**

$$\begin{aligned}\nabla^2 V &= \nabla \cdot (\nabla V) \\ &= \text{divergence of gradient of } V\end{aligned}$$

**5. (b)**Let the unit vector be given by  $\vec{a}_R$ .Now,  $\vec{R} =$  Difference of two vectors

$$= r\vec{a}_r - h\vec{a}_z$$

$$\therefore \text{Unit vector, } \vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$$

**6. (a)**

A phasor is always a vector quantity.

**7. (b)**

Gradient of a scalar;

 $\nabla A =$  maximum rate of change of scalar  $A$  with respect to given coordinates system.**8. (a)**

Both assertion and reason are true and reason is the correct explanation of assertion. Reason is the physical interpretation of divergence.

**10. (b)**The vector  $\vec{A}$  is given as

$$\begin{aligned}\vec{A} &= (0-2)\vec{a}_x + [-2-(-4)]\vec{a}_y + (0-1)\vec{a}_z \\ &\quad \begin{array}{c} x \bullet \xrightarrow{\vec{A}} \bullet y \\ (2, -4, 1) \qquad \qquad \qquad (0, -2, 0) \end{array} \\ &= -2\vec{a}_x + 2\vec{a}_y - \vec{a}_z\end{aligned}$$

**11. (c)**The vector field  $\vec{A}$  will be irrotational, if  $\nabla \times \vec{A} = 0$ .

$$\begin{aligned}\text{Now, } \nabla \times \vec{A} &= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \\ &= \left[ \frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz) \right] \vec{a}_x \\ &\quad + \left[ \frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz) \right] \vec{a}_y \\ &\quad + \left[ \frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz) \right] \vec{a}_z\end{aligned}$$

$$= [x - x] \vec{a}_x + [y - y] \vec{a}_y + [z - z] \vec{a}_z$$

$$= 0$$

Hence,  $\vec{A}$  is irrotational.

The vector field  $\vec{A}$  will be solenoidal, if  $\nabla \cdot \vec{A} = 0$

Here,

$$\nabla \cdot \vec{A} = \left( \vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z} \right) \cdot (yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z)$$

$$= \vec{a}_x \cdot \vec{a}_x \frac{\partial}{\partial x} (yz) + \vec{a}_y \cdot \vec{a}_y \frac{\partial}{\partial y} (xz) + \vec{a}_z \cdot \vec{a}_z \frac{\partial}{\partial z} (xy)$$

$$= 0 + 0 + 0 = 0$$

Hence,  $\vec{A}$  is solenoidal.

**12. (a)**

Given,  $\vec{A} = \frac{1}{\sqrt{x^2 + y^2}} \vec{a}_x$

$$\therefore \nabla \cdot \vec{A} = \frac{\partial}{\partial x} (A_x) + \frac{\partial}{\partial y} (A_y) + \frac{\partial}{\partial z} (A_z)$$

$$= \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{x^2 + y^2}} \right) + 0 + 0$$

$$= \frac{\partial}{\partial x} (x^2 + y^2)^{-1/2}$$

$$= -\frac{1}{2} (x^2 + y^2)^{-3/2} \cdot 2x$$

$$= \nabla \cdot \vec{A} = -\frac{x}{\sqrt{(x^2 + y^2)}(x^2 + y^2)}$$

$$\text{Now, } (\nabla \cdot \vec{A})_{2,2,0} = -\frac{2}{\sqrt{(2^2 + 2^2)} \cdot (2^2 + 2^2)}$$

$$= -\frac{2}{\sqrt{8} \cdot 8} = -0.0884$$

**13. (b)**

Given,  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$$\therefore \vec{r} \times \vec{i} =$$

$$(x\vec{i} + y\vec{j} + z\vec{k}) \times \vec{i} = -y\vec{k} + z\vec{j}$$

Also,  $\vec{i} \times (\vec{r} \times \vec{i}) =$

$$\vec{i} \times (-y\vec{k} + z\vec{j}) = \vec{j}y + z\vec{k}$$

Similarly,  $\vec{j} \times (\vec{r} \times \vec{j}) = \vec{i}x + \vec{k}z$

and  $\vec{k} \times (\vec{r} \times \vec{k}) = \vec{i}x + \vec{j}y$

Thus,  $\vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k})$

$$= 2(x\vec{i} + y\vec{j} + z\vec{k}) = 2\vec{r}$$

**14. (d)**

Since vector  $\vec{V}$  is solenoidal, therefore

$$\nabla \cdot \vec{V} = 0$$

$$\therefore \left[ \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right] \cdot \left[ \vec{i}(x + 3y) + \vec{j}(y - 2x) + \vec{k}(x + bz) \right] = 0$$

or,

$$[1 + 1 + b] = 0 \text{ or } b = -2$$

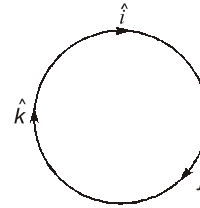
**15. (c)**

Reason is a statement of stroke's theorem not that of Gauss's divergence theorem.

**16. (c)**

A scalar field  $V$  is harmonic, if  $\nabla^2 V = 0$ . Hence, option (c) is not correct.

**17. (c)**



Option (c) is not correct because cross product of two unlike vectors is a third unit vector having positive sign for normal rotation and negative for reverse rotation while cross product of two like unit vectors is zero.

**18. (d)**

Taking the curl, we have:

$$\nabla \times \vec{v} = \nabla \times \vec{\omega} \times \vec{r} \text{ (Since } \vec{v} = \vec{\omega} \times \vec{r} \text{)}$$

$$\text{or, } \nabla \times \vec{v} = \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix}$$

$$= \left[ \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right] \times [\vec{i}(\omega_y z - \omega_z y) + \vec{j}(\omega_z x - \omega_x z) + \vec{k}(\omega_x y - \omega_y x)]$$

$$= (\vec{i} \times \vec{j}) \frac{\partial}{\partial x} (\omega_z x - \omega_x z) + (\vec{i} \times \vec{k}) \frac{\partial}{\partial x} (\omega_x y - \omega_y x)$$

$$+ (\vec{j} \times \vec{i}) \frac{\partial}{\partial y} (\omega_y z - \omega_z y)$$

$$+ (\vec{j} \times \vec{k}) \frac{\partial}{\partial y} (\omega_x y - \omega_y x) + (\vec{k} \times \vec{i}) \frac{\partial}{\partial z} (\omega_y z - \omega_z y) + (\vec{k} \times \vec{j}) \frac{\partial}{\partial z} (\omega_z x - \omega_x z)$$

$$= \vec{k}(\omega_z - 0) - \vec{j}(0 - \omega_y) - \vec{k}(0 - \omega_z)$$

$$+ \vec{i}(\omega_x - 0) + \vec{j}(\omega_y - 0) - \vec{i}(0 - \omega_x)$$